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Elastic Behavior Analysis of Polymer Melt Extruding through Capillary with An Additional Sinusoidal Vibration

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Summary

A novel rheological measuring apparatus was designed by the authors, which introduced an additional sinusoidal vibration in parallel on the extruding direction of polymer melt and was called as Constant Velocity Dynamic Capillary Rheometer in this article. After introducing different melt extrusion speeds and different intensities of vibration, the rheological measuring experiments for a LDPE melt were done respectively by using above-mentioned apparatus, and a mathematical model for the primary normal stress difference of polymer melt under the action of vibration was set up accordingly. Then, the change laws of die swell ratio and those of the primary normal stress difference under different frequencies and amplitudes of vibration were obtained. Compared with the stable state extrusion, after introducing an additional sinusoidal vibration, the die swell ratio and the primary normal stress difference of polymer melt are all decreased obviously.

Introduction

Die swell is one of the typical elastic behaviors of polymer melt, and the normal stress difference is a theoretic characterization of above elastic behaviors [1]. Die swell may result in the change of the final dimension of products. In order to realize the precise control for product quality and the correct design for mould structure, it is necessary to investigate the mechanism of die swell. After introducing a vibration force field into the extruding process of polymer melt, the phenomenon of die swell changes accordingly [2-5]. Researching the relationship between vibration force field with die swell or normal stress difference possesses an important guide meaning for optimizing the technological and structural parameters of dynamic forming equipment, and also for getting the products with good performance.

Theory

The Constant Velocity Dynamic Capillary Rheometer [6-10] superimposes a sinusoidal vibration in parallel on the extruding direction of polymer melt through a capillary, as shown in Fig. 1. Assuming that the length of capillary is L, the radius is

R, the length of entire development region is L'. For convenience, the cylinder coordinate system r, θ, z was adopted, in which the direction of flow velocity of melt is on z direction, that of velocity gradient is on r direction, and the neutral direction is on θ direction, as shown in Fig. 2. When theoretically analyzing the dynamic flow of polymer melt within capillary, it is also assumed that the melt plays

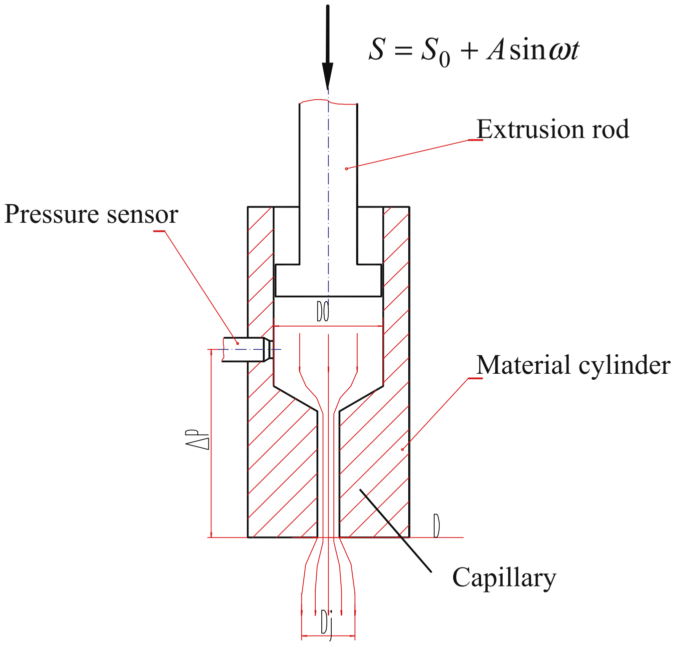

Fig. 1. Schematic drawing of dynamic extrusion of capillary

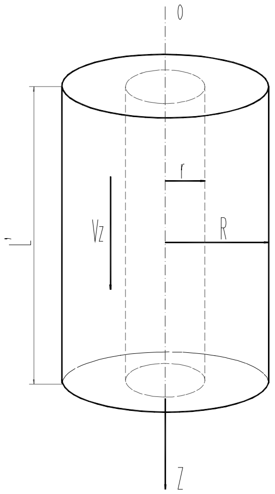

Fig. 2. Full developed flow of polymer melt within capillary

isothermal, full developed and axial stratified flow, and is incompressible, there is no slipping on the inside wall of capillary $(r = R)$ and the gravitational force is omitted. Under an additional sinusoidal vibration, the extrusion rod plays a sinusoidal displacement vibration, thus the instant volumetric flow rate $Q(t)$ of the capillary may be proposed as: [6-10]

$$Q(t) = \overline{Q}\cdot(1+\varepsilon_q\cos\omega t) \qquad \textbf{(1)}$$

where $\overline{Q}$ is the average volumetric flow rate of the capillary, ω is the circular frequency of vibration $(\omega = 2\pi f)$, $\overline{Q}\cdot\varepsilon_q\cos\omega t$ is the pulsating part of the volumetric flow rate, ε_q is the pulsating amplitude value coefficient of the volumetric flow rate.

Then, the average velocity $V(t)$ of polymer melt within capillary on the section may be supposed as: [6]

$$V(t) = \overline{V}\cdot(1+\varepsilon_\upsilon\cos\omega_\upsilon t) \qquad \textbf{(2)}$$

where $\overline{V}$ is the mean flow velocity of melt within capillary, $\overline{V}\cdot\varepsilon_\upsilon\cos\omega_\upsilon t$ is the pulsating part of the flow velocity, ε_υ is the pulsating amplitude value coefficient of the flow velocity, ω_υ is the pulsating frequency of the flow velocity.

Similarly, the instant pressure drop of the capillary may also be proposed as: [6-10]

$$\Delta p(t) = \overline{\Delta p}\cdot[1+\varepsilon_p\cos(\omega t+\varphi)] \qquad \textbf{(3)}$$

where $\overline{\Delta p}$ is the average pressure drop of the capillary, $\overline{\Delta p}\cdot\varepsilon_p\cos(\omega t+\varphi)$ is the pulsating part of the pressure drop, ε_p is the pulsating amplitude value coefficient of the pressure drop, φ is the phase-difference of the stress and the strain.

The reciprocal D/D_j of the instant die swell ratio of polymer melt within capillary can be also proposed as: [6]

$$\frac{D}{D_j} = \frac{1}{\overline{B}}\cdot(1+\varepsilon_B\cos\omega_B t) \qquad \textbf{(4)}$$

where $\overline{B}$ is the mean die swell ratio of melt within capillary, $\frac{1}{\overline{B}}\varepsilon_B\sin\omega_B t$ is the pulsating part of the reciprocal of die swell ratio, ε_B is the pulsating amplitude value coefficient of the reciprocal of die swell ratio, ω_B is the pulsating frequency of the reciprocal of die swell ratio (here $\omega_B = \omega_q$).

Then the time-average value of the shear stress [7] and shear rate [8] at capillary wall under an additional sinusoidal vibration can be respectively obtained as:

$$\begin{aligned}\overline{\tau_w(t)} &= \frac{1}{T}\int_0^T \tau_w(t)\cdot dt\\ &= \frac{1}{T}\int_0^T [\frac{R}{2(L+N_B\cdot R)}\cdot\Delta p(t) - \frac{\rho}{2\pi R}\cdot\overline{Q}\cdot\varepsilon_q\cdot\omega\cdot\sin\omega t]\\ &= \frac{R}{2(L+N_B R)}\cdot\overline{\Delta p}\\ &= a\cdot\overline{\Delta p}\end{aligned} \qquad \textbf{(5)}$$

$$\overline{\dot{\gamma}_w(t)} = \frac{\overline{Q}}{\pi R^3}\cdot[3+\frac{a^2\cdot\overline{\Delta p}^2\cdot\varepsilon_q\cdot\varepsilon_p\cdot\cos\varphi}{a^2\cdot\overline{\Delta p}^2\cdot{\varepsilon_p}^2+2a\cdot b\cdot\overline{\Delta p}\cdot\overline{Q}\cdot\varepsilon_q\cdot\varepsilon_p\cdot\omega\cdot\sin\varphi+b^2\cdot\overline{Q}^2\cdot{\varepsilon_q}^2\cdot\omega^2}] \qquad \textbf{(6)}$$

The conservation of mass and momentum laws are applied in the solution of die swell of polymer melt extruding through a capillary under the action of vibration. The control volume range is from the exit plane of capillary (see section 1 in Fig. 3) to another plane (see section 2 in Fig. 3).

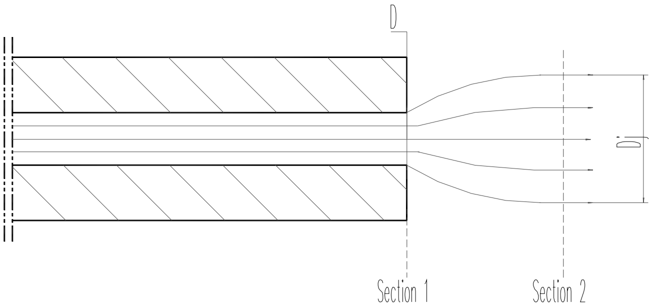

Fig. 3. Schematic drawing of die swell analysis of capillary

It may be known from previous analysis of authors [6-10] that the time-average value of the primary normal stress difference of polymer melt at capillary wall under an additional sinusoidal vibration is as follow:

$$
\begin{aligned}
&\overline{\tau_{zz}(R,t)-\tau_{rr}(R,t)}=\rho\overline{V}^{2}\cdot\\
&[1+\frac{\varepsilon_{v}^{2}}{2}+\frac{a^{2}\cdot\overline{\Delta p}^{2}\cdot\varepsilon_{p}\cdot\varepsilon_{v}\cdot\cos\varphi}{a^{2}\cdot\overline{\Delta p}^{2}\cdot\varepsilon_{p}^{2}+2a\cdot b\cdot\overline{\Delta p}\cdot\cdot\overline{Q}\cdot\varepsilon_{q}\cdot\varepsilon_{p}\cdot\omega_{q}\cdot\sin\varphi+b^{2}\cdot\overline{Q}^{2}\cdot\varepsilon_{q}^{2}\cdot\omega_{q}^{2}}]\\
&-\rho\frac{\overline{V}^{2}}{\overline{B}^{2}}\cdot\\
&[1+\frac{\varepsilon_{B}^{2}}{2}+\frac{a^{2}\cdot\overline{\Delta p}^{2}\cdot\varepsilon_{p}\cdot\varepsilon_{B}\cdot\cos\varphi}{a^{2}\cdot\overline{\Delta p}^{2}\cdot\varepsilon_{p}^{2}+2a\cdot b\cdot\overline{\Delta p}\cdot\overline{Q}\cdot\varepsilon_{q}\cdot\varepsilon_{p}\cdot\omega_{q}\cdot\sin\varphi+b^{2}\cdot\overline{Q}^{2}\cdot\varepsilon_{q}^{2}\cdot\omega_{q}^{2}}]
\end{aligned}
\qquad \textbf{(7)}
$$

where $a=\frac{R}{2(L+N_{B}\cdot R)}$, $b=\frac{\rho}{2\pi R}$, ρ is the melt density, N_{B} is the revised factor of pressure drop under vibrating force field.

These parameters in equation (7) can be all obtained by using the Constant Velocity Dynamic Capillary Rheometer, with real-time collecting the dynamic rheological data of polymer melt, such as the instant value of die swell, the instant entry pressure of capillary, the instant vibrating displacement of extrusion rod, the phase difference between the waves of vibrating displacement and entry pressure of capillary, and so on, then undergoing a time-domain analysis or a frequency-domain analysis for such dynamic rheological data. [6-10]

Experiment

A LDPE was used for the dynamic rheological experiment, using the Constant Velocity Dynamic Capillary Rheometer designed by the authors. The falling velocities of extrusion rod were set to be at five rates, i.e. 600, 700, 800, 900, 1000 mm/min respectively; the vibrating frequency of extrusion rod was set to be 0, 6, 8, 10, 12, 18 Hz respectively; the nominal amplitude of vibration was calculated to be 0, 0.05, 0.075, 0.1, 0.125, 0.15 mm; in the experiment, the die of the capillary had various ratios of length to diameter (L/D =10, 15, 20). The melt temperature of LDPE was set to be 165°C.

With processing the above-mentioned experimental data and making use of the characterization formula of the primary normal stress difference of polymer melt at capillary wall under an additional sinusoidal vibration, the feature curves which response the elastic behavior of polymer melt under the action of vibration may be obtained respectively. Such curves contain: (1) effects of vibration on the die swell ratio of polymer melt within capillary; (2) effects of vibration on the primary normal stress difference at capillary wall.

Results and discussion

1. Effects of Vibration on the Die Swell Ratio of Polymer Melt

The die swell ratio of melt, namely, the ratio of the extrudate diameter to the diameter of die, can express the amount of recoverable elastic energy stored in the flow materials. It is also dependent upon the concrete material, the melt temperature, the shear rate and the geometric feature of a die. After introduced an additional sinusoidal vibration in the rheological experiment of a LDPE, it has been found that the die swell ratio B has a direct behaving on the strength parameters of vibration (i.e. the amount of frequency and amplitude).

Fig. 4 and Fig. 5 are respectively the relationship curves between the die swell ratio B of LDPE melt and the vibrating frequency f during two falling velocity of extrusion rod (i.e. u_0 =600 and 1000 mm/min). The three curves in each Fig. are respectively the alteration of B with f at three preset amplitudes (when A =0mm, being stable state extrusion). The constant temperature of melt T=165°C, L/D=20.

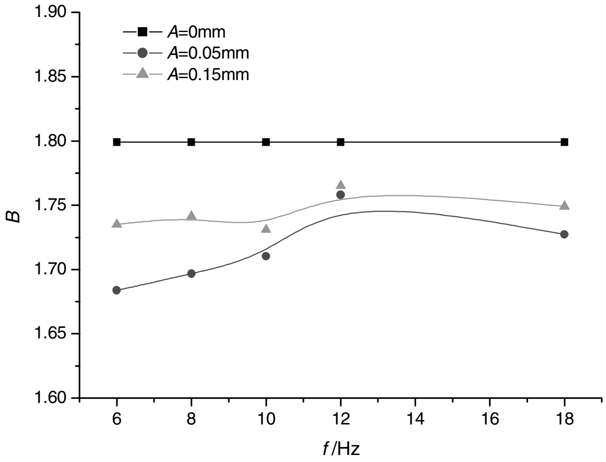

Fig. 4. Relation between die swell ratio and frequency while u_0 =600mm/min

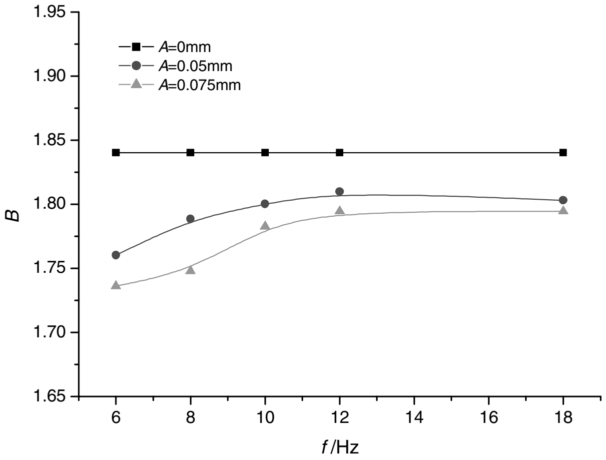

Fig. 5. Relation between die swell ratio and frequency while u_0 =1000mm/min

It can be seen from Fig. 4 and Fig. 5 that: (1) as compared with the stable state extrusion, the die swell ratio of LDPE melt diminishes obviously after introducing a vibration; (2) under the same vibrating amplitude, the reducing degree of die swell ratio appears as a non-linear decreasing tendency with an increase in vibrating frequency (the vibrating frequency range: 0-18Hz).

The reasons leading to die swell are two respects [1]: One is the entry effect. The polymer melt will store elastic energy since the strong tensile deformation occurring in the entry area of die, if the elastic energy does not relax completely in die, they will continuously relax at the exit of die, that is die swell. Two is the shear flow. When polymer melt flows through capillary, elastic deformation will occur due to the effect of shear stress and normal stress difference, which in turn exhibit the relaxation. When the length-diameter ratio (L/D) of die is small, the entry effect is major, and the die swell is chiefly caused by tensile flow. If the length-diameter ratio (L/D) is larger than 20, the die swell is primarily arisen by shear flow, that is the result of shear stress and normal stress difference.

The introduction of vibrating force field improved the fluidity of polymer melt and decreased the apparent viscosity [10], under the identical flow rate of extrusion, the shear stress and the normal stress difference within die were diminished [6,7], and then reducing the die swell. According to the viewpoint of molecular structure, the introduction of vibrating force field promotes the untying tangle and the orientation of molecular chain, and shortens the relaxation time of melt, thereby lowers the amount of recovery deformation at the exit of die.

2. Effects of Vibration on Primary Normal Stress Difference at Capillary Wall

When depicting the stress condition of viscoelastic fluid in simple shear flowing field, the normal stress difference (including the primary normal stress difference and the second normal stress difference) which characters the elastic behavior of polymer melt is adopted [1]. It has been proved that in stable stale the second normal stress difference is far less than the primary normal stress difference, and it is a negative value. Therefore, we take the primary normal stress difference as the measurement of elastic behavior of polymer fluid, and experimentally researching the effects of vibration on the primary normal stress difference of capillary wall.

Fig. 6 and Fig. 7 are respectively the relationship curves between the primary normal stress difference of melt $N_1(f,A)$ and the vibrating frequency f under two falling velocity of extrusion rod (i.e. u_0 =600 and 800 mm/min). The four curves in each Fig. are respectively the change of $N_1(f,A)$ with f at four preset vibrating amplitudes (when A =0mm, being stable state extrusion). The constant temperature of melt T=165°C, L/D=20.

It is observed from Fig. 6 and Fig. 7 that: (1) as compared with the stable state extrusion, after introducing a vibration, the primary normal stress difference at capillary wall $N_1(f,A)$ decreases obviously; (2) under the same vibrating amplitude, the reducing degree of primary normal stress difference displays a nonlinear lowering trend with increasing the vibrating frequency (the vibrating frequency range: 0-18Hz).

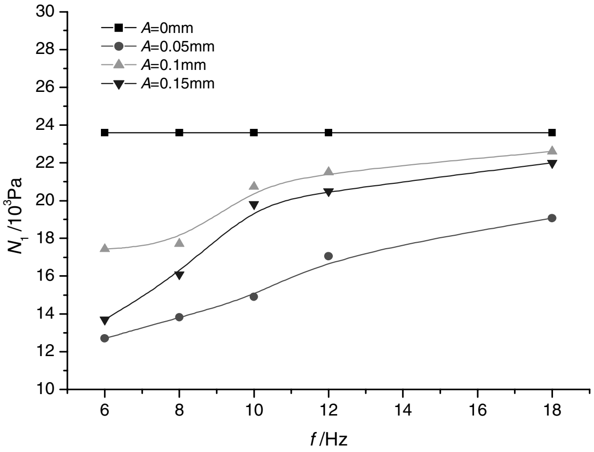

Fig. 6. Relation between primary normal stress difference and frequency while u_0=600mm/min

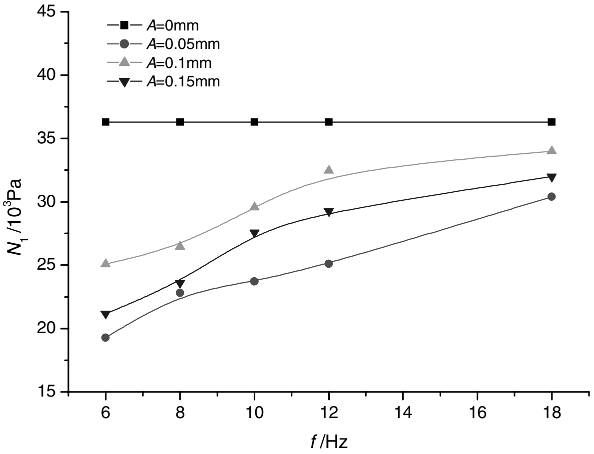

Fig. 7. Relation between primary normal stress difference and frequency while u_0=800mm/min

Conclusions

With making use of the Constant Velocity Dynamic Capillary Rheometer, the dynamic rheological experiments were done for a LDPE, getting the alteration laws of the primary normal stress difference at capillary wall and the die swell ratio of melt under various frequencies and amplitudes of an additional sinusoidal vibration. As compared with the stable extrusion, it was found that both the die swell ratio and the primary normal stress difference decrease obviously after introducing a vibration; under an identical vibrating amplitude, the reducing degree of the die swell ratio or the primary normal stress difference appears as a tendency of nonlinear lowering with an enhancement in vibrating frequency; under the same vibrating frequency, the reducing degree of die swell ratio or the primary normal stress difference also vary with the alteration of vibrating amplitude.

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